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ANALOGY BETWEEN THE RESPONSE OF A TURBULENT JET AND A PENDULUM WITH A RANDOMLY VIBRATING SUSPENSION AXIS TO PERIODIC EXCITATION

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Experimental dependences, which illustrate the influence of the frequency and amplitude of the acoustic action on the aerodynamic characteristics of turbulent jets and mixing layers, are compared to the corresponding dependences obtained for a pendulum with a vibrating suspension axis. The similarity of them is noted. It is essential that the similarity indicated is based on the commonness of the nature of the phenomena occurring in these systems under the periodic action rather than on the similarity of the equations of motion in both cases.

Acoustic and periodic actions on turbulent jets turned out to be an efficient means of controlling their aerodynamic and acoustic characteristics [1]. This control is due to the strong influence of periodic disturbances on the process of formation, development, interaction, and destruction of large-scale structures in the mixing layer of the initial portion of the jet. Two effects which occur under these conditions, i.e., enhancement of mixing at low frequencies of harmonic acoustic disturbances (the Strouhal number St = fd/u_0 = 0.2–0.6) and weakening of mixing at high frequencies (St = 2–6), have been established.

In this case, as the level of action at low frequencies increases, the effect enhances to the point of saturation, and at high frequencies an increase in the level of action leads to a weakening of mixing that reaches its optimum, whereupon further increase in the level results in the opposite effect, i.e., instead of the weakening of mixing, its enhancement occurs, as is the case in low-frequency excitation. The quest for enhancement of the acoustic action on turbulent jets led one to attempt to use an acoustic signal with several frequencies, e.g., the basic frequency and its subharmonic with a fixed shift of phases between them.

The effects obtained can be described within the framework of mathematical modeling of the action of periodic disturbances on turbulent jets. For this purpose different methods are used, viz., direct numerical modeling of turbulent flow in the initial portion of an axisymmetric jet on the basis of nonstationary Euler equations in subsonic flow, modeling of turbulent jets on the basis of the method of discrete vortices, modeling on the basis of nonstationary Reynolds equations closed by the differential model of turbulence, and, finally, on the basis of generalized Reynolds equations obtained in the case of representation of flow parameters in the form of three components — stationary, coherent large-scale, and random small-scale [1].

In what follows, we consider a different approach to description of the effects mentioned. The approach is based on the hypothesis [2–4] according to which turbulence in open flows, in particular, in jets, is caused by the presence of weak random disturbances (noise) which are sharply enhanced due to instability and convert the system to a qualitatively new state. This change is given the name noise-induced phase transition. Similarly to many other complex vibrational phenomena (e.g., fibrillation in the human heart, which in 1928 was explained by the Dutch researchers Van der Pol and Van der Marc on the basis of consideration of an electric model consisting of three simple generators), the phenomena of noise-induced phase transition can be studied with the example of the simplest model where such a transition is possible [5]. It is essential that the equations of this model can fundamentally differ from the equations of the studied system. The reason is the universal character of the laws of vibration theory. We took a pendulum with a randomly vibrating axis of suspension as such a model. It turned out that the excitation of vibrations in this pendulum is surprisingly similar to the excitation of turbulence in jets. Moreover, the addition of a periodic

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component to the random vibration of the axis of the pendulum suspension made it possible to model the processes of control of turbulence by an acoustic action. The relations obtained appeared to be very similar to the corresponding relations for a jet.

The equation of vibrations of the above-mentioned pendulum can be written in the form

$$\ddot{\varphi} + 2\beta\,(1 + 2\alpha\dot{\varphi}^2)\,\dot{\varphi} + \omega_0^2\,[1 + \xi_1\,(t) + A\cos\omega t]\,\sin\varphi = \xi_2\,(t)\,, \tag{1}$$

where $2\beta(1 + 2\alpha\dot{\varphi}^2)\dot{\varphi}$ is the quantity proportional to the moment of friction forces, ω_0 is the frequency of small eigenmodes of the pendulum, β and α are the coefficients of linear and nonlinear friction, respectively, $\xi_1(t) + A$ cos ωt is a quantity proportional to the acceleration of the axis of the pendulum suspension and is the sum of the comparatively wide-band random process $\xi_1(t)$ with non-zero spectral density at the frequency $2\omega_0$ (which we denote by κ and call the intensity of random vibration) and the harmonic component A cos ωt, and $\xi_2(t)$ is the additive component of the random vibration of the suspension axis arising in the case where the direction of vibration differs from the vertical. Analytical and numerical solutions of Eq. (1) at $A = 0$ showed that for a low intensity of the vibration of the suspension axis the pendulum executes small random vibrations near the equilibrium position which are due to the additive component $\xi_2(t)$ (if this component is absent, the pendulum is at rest). When the intensity of random vibrations of the suspension axis exceeds some critical value κ_{cr}, the swing of the pendulum sharply increases — even at $\xi_2(t) = 0$ the dispersion of its rotation angle becomes nonzero. This phenomenon has come to be known as noise-induced phase transition. It turned out that near the threshold of this excitation of pendulum vibrations (at $\xi_2(t) = 0$) we have the property of intermittency of special type, when during long time intervals the pendulum vibrates near its equilibrium position and rather strong bursts (so-called "laminar" and "turbulent" phases) appear from time to time. With distance from the excitation threshold the length of the laminar phases decreases, whereas that of the turbulent phases increases, which results in the disappearance of the laminar phases. In this case, the dispersion of the vibrations increases monotonically. In the presence of the additive component $\xi_2(t)$, intermittency occurs below the threshold and is weak.

It was found that additional weak harmonic vibrations of the suspension axis strongly affect the processes of excitation of the pendulum; low-frequency vibrations increase the intensity of noise-induced vibrations and reduce the threshold of their excitation, whereas high-frequency ones, on the contrary, suppress noise-induced vibrations and increase this threshold. As in the case of the high-frequency acoustic action on a jet at small amplitudes of the high-frequency component of the vibration of the axis of the pendulum suspension it virtually does not affect the existing vibrations. With increase in the amplitude of the high-frequency components of the vibration the intensity of the noise-induced vibrations of the pendulum first decreases to a certain minimum value, which is the smaller the larger the frequency of vibration, and then begins to increase; beginning with a certain value of the intensity, the opposite effect occurs — in the presence of the high-frequency harmonic component of the vibration the intensity of pendulum vibrations becomes higher than in its absence. In this case, we also have a total analogy with the results of the high-frequency acoustic action on a jet.

Within the framework of this approach we obtained dependences which illustrate the influence of the frequency and amplitude of the harmonic components of vibration of the suspension axis on the dispersion of the rotation angle of the pendulum. They are surprisingly similar to the corresponding dependences obtained in the case of low- and high-frequency acoustic actions exerted on turbulent jets and mixing layers with different frequencies and amplitudes.

In what follows, we give a comparison of the experimental dependences for acoustically excited turbulent jets and mixing layers on the frequency and the level of excitation and the analogous calculated dependences for noise-induced vibrations of the pendulum with a randomly vibrating suspension axis.

The results of the experiments and calculations are presented in Fig. 1–4, each of which gives the data on acoustic excitation of turbulent jets on the left and the data of calculation for the pendulum with a vibrating suspension axis on the right. The dependences of Fig. 1 describe the above-mentioned effect of saturation with increase in the amplitude in the case of low-frequency action [6].

The data of Fig. 2 illustrate the influence of the amplitude of vibrations in both cases under high-frequency action. Here, with increase in the level of acoustic action the fluctuations of velocity in the jet first decrease, reach a

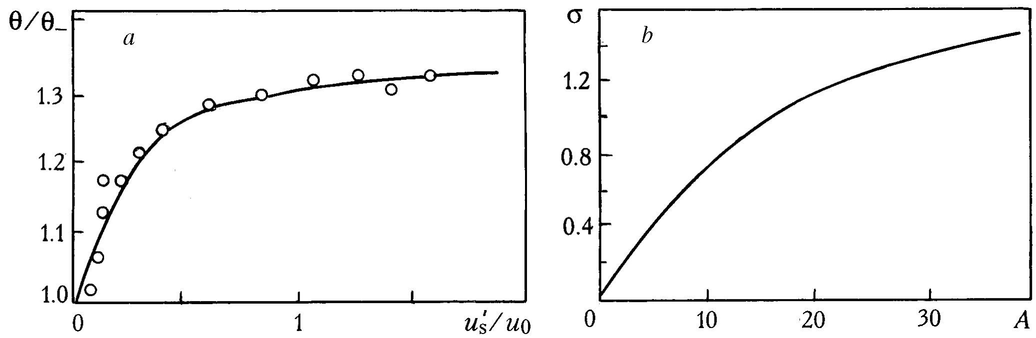

Fig. 1. Dependence of the momentum thickness of the jet in the cross section x/d = 9 on the level of low-frequency (St = 0.5) acoustic excitation u_s'/u_0 [6] (a) and the dependence of the pendulum parameter σ on the amplitude A of low-frequency ($\omega = 1$) harmonic vibrations (b); κ/κ_{cr} = 2.23. u_s'/u_0, %.

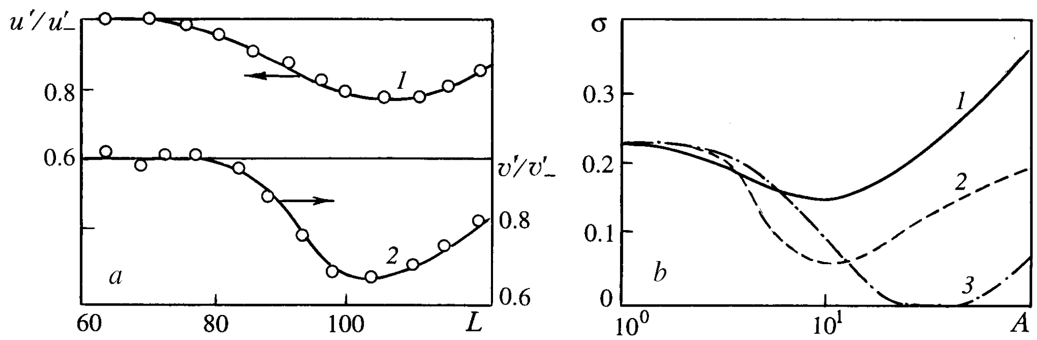

Fig. 2. Change in the longitudinal and transverse velocity fluctuations on the jet axis u'/u'_- and v'/v'_- at the point x/d = 8 as a function of the level of sound pressure L of the high-frequency (St = 2.35) acoustic effect [1] (a) and the dependence of the parameter σ on the amplitude A of pendulum vibrations at the frequencies $\omega = 3.5$ (1), 6 (2), and 11 (3) (b); κ/κ_{cr} = 5.6. L, dB.

minimum, and then begin to increase; we observe a tendency toward the opposite effect when, with increase in the level of excitation, the high-frequency action leads to an increase in the velocity fluctuations rather than to their decrease [1]. In the dependences presented for longitudinal acoustic irradiation of the jet, the minimum of u'/u'_- and v'/v'_- is reached at L = 104–108 dB, which corresponds to values of u_s'/u_0 = 0.075–0.12%. The above-mentioned tendency for the jet is also observed [7] at high-frequency irradiation of a turbulent mixing layer (Fig. 3). Calculations for a pendulum at high frequencies of the vibration action mainly confirm the character of the influence of the amplitude obtained in experiments with jets under acoustic action. However, in this case, a significant difference is revealed, viz., vibrations are totally suppressed at large values of the amplitude. This difference is eliminated if on the right-hand side of Eq. (1) one assumes that the function $\xi_2(t)$ is nonzero. This corresponds to the action of both vertical and horizontal vibrations on the suspension axis.

Particular emphasis must be placed on the dependences presented in Fig. 4. To enhance the effect of acoustic action on turbulent jets and mixing layers, in some cases use is made of two-frequency acoustic excitation at the basic frequency and its subharmonic with different phase shifts between them:

$$u' = a_1 \cos(2\pi f t) + b_1 \cos(\pi f t + \psi)\,,$$

where f and $f/2$ are the basic frequency and its subharmonic and ψ is the phase shift between them.

Experiments showed [8] that in this case at low frequencies and the corresponding phase shift one manages to considerably enhance the intensity of mixing, whereas at high frequencies the two-frequency action does not give any differences from the case of the one-frequency action at all values of the phase shift. The data presented in Fig.

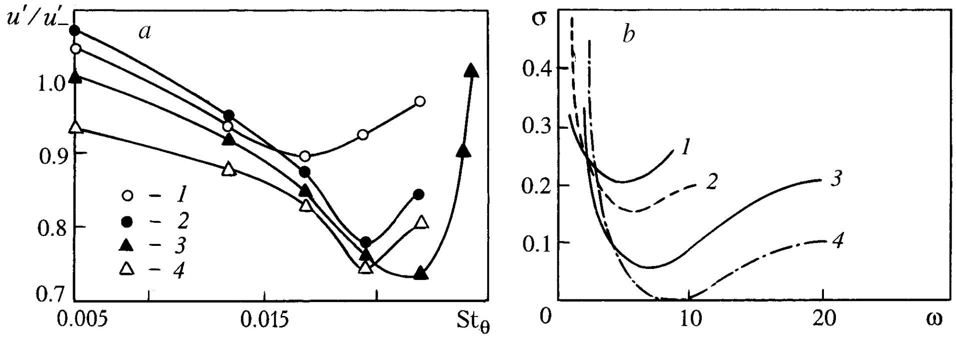

Fig. 3. Dependence of the longitudinal velocity fluctuations in the turbulent mixing layer [7] in the cross section x/θ_0 = 200 on the Strouhal number of acoustic excitation $\mathrm{St}_\theta = f\theta_0/u_0$ at u_s'/u_0 = 0.5% (1), 2.5 (2), 3.5 (3), and 4.5 (4) (a) and the similar dependences for the pendulum $\sigma = F(\omega)$ at values of the amplitude of vibrations of A = 2.5 (1), 5 (2), 10 (3), and 20 (4) (b); κ/κ_{cr} = 5.6.

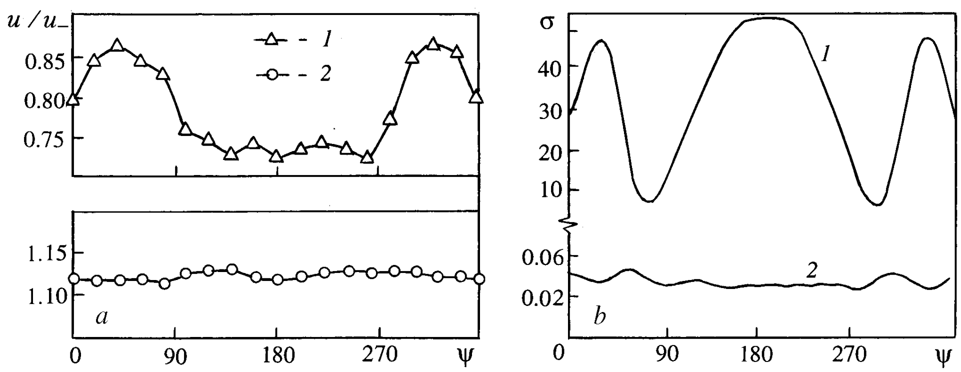

Fig. 4. Change in the mean velocity on the jet axis in the cross section x/d = 8 in its two-frequency acoustic excitation at the basic frequency and its subharmonic for St = 0.8 and 0.4 (1) and St = 7.8 and 3.9 (2) as a function of the phase shift ψ between them [8] (a) and dependences for the pendulum $\sigma = F(\psi, \omega)$ at low frequencies (ω = 0.3 and 0.15) (1) and high frequencies (ω = 11 and 5.5) (2) (b); κ/κ_{cr} = 5.6. ψ, deg.

4a correspond to transverse acoustic irradiation of the jet at the basic frequency and its subharmonic with a level of sound pressure of L = 122 dB, which for an outflow velocity of u_0 = 20 m/sec corresponds to v'/u_0 = 0.59%. The data of Fig. 4 show a similar character of the influence of the phase shift in both cases. The results for the pendulum (Fig. 4b) correspond to the same values of the amplitudes of the harmonic components at the basic frequency and its subharmonic A = 5.

For the purity of the effect, pendulum vibrations were calculated at $\xi_2(t)$ = 0 and different values of the intensity of the random vertical component of the vibration of the suspension axis. The remaining parameters were fixed and equal to $\omega_0 = 1$, $\beta = 0.1$, and α = 100.

The comparisons presented confirm the validity of the analogy indicated in the title of the paper.

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NOTATION

x, longitudinal coordinate; d, diameter of the outlet cross section of the nozzle; u_0, velocity of outflow of the jet at the nozzle cut; u' and v', root-mean-square values of longitudinal and transverse velocity fluctuations; u_s' and v_s'', root-mean-square values of velocity fluctuations in a sound wave; L, level of sound pressure, dB; f, frequency of acoustic excitation; θ, momentum thickness; t, time; φ, angular deviation of the pendulum relative to the equilibrium position; $\langle\varphi^2\rangle$, dispersion; $\sigma = (\langle\varphi^2\rangle)^{1/2}$; A, amplitude of the harmonic component of the vertical vibration of the axis of the pendulum suspension; ω, frequency of the harmonic component of the vibration of the axis of the pendulum suspension; κ and κ_{cr}, intensity of the random vibration of the axis of the pendulum suspension and its critical value, respectively. Subscripts: minus, jet parameter in the absence of acoustic excitation; 0, at the nozzle cut; s, velocity; cr, critical.

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